LEBANESE AMERICAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS
MTH 206 - CALCULUS 3
FINAL EXAM – SPRING 2012

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Duration: 120 minutes

Name: **ID#:**

- **This exam consists of 12 pages and 11 problems.**
- **Answer the questions below on the space provided. You can use the back pages for scratch or for more space for your answers. Please specify.**
- **Make sure you justify all your answers.**

Question Number	**Grade**
1. 8%	
2. 8%	
3. 8%	
4. 8%	
5. 8%	
6. 8%	
7. 12%	
8. 9%	
9. 8%	
10. 8%	
11. 15%	
TOTAL	

Problem 1: (8%)

If $f(x, y, z)$ is a differentiable function, $x = r - s$, $y = s - t$ and $z = t - r$, show that

$$f_r + f_s + f_t = 0$$

Problem 2: (8%)

Find a direction in which the function $f(x, y) = xe^{-4y^2}$ incurs no change at the point $P(1, 0)$.

Problem 3: (8%)

Find the points on the surface $(y + z)^2 + (z - x)^2 = 16$ where the normal line is parallel to the $y - z$ plane.

Problem 4: (8%)

Let $f(x, y) = e^{10x-5y}$. Using linearization, approximate $f(0.05, -0.03)$.

Problem 5: (8%)

If the sum of squares of two real numbers is 10, find the maximum of their product.

Problem 6: (8%)

Find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$.

Problem 7: (12%)

At a point P, the velocity and acceleration of a particle moving in the plane are $\vec{v} = 3\vec{i} + 4\vec{j}$ and $\vec{a} = 5\vec{i} + 15\vec{j}$. Find the curvature of the particle's path at the point P.

Problem 8: (9%)

Name formulas and theorems that connect the following. Make sure you mention the relevant conditions in each case (you may need more than one step to go from one integral to another):

(a) Line integral to surface integral

(b) Line integral to triple integral

(c) Surface integral to double integral

Problem 9: (8%)

Use the **most efficient method** to compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = \frac{1}{y}\vec{i} + (\frac{1}{z} - \frac{x}{y^2})\vec{j} - \frac{y}{z^2}\vec{k},$$

and $\mathcal{C}$ is the circle in the $y - z$ plane centered at $(0, 2, 1)$ with radius 1.

Problem 10: (8%)

Use the **most efficient method** to find the outward flux of the vector field

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$$

across the boundary of the cube bounded by $x = \pm 1, y = \pm 1$ and $z = \pm 1$.

Problem 11: (15%)

(a) Let $f(x)$ be the periodic function of period $T = 2$ defined as

$$f(x) = |x| \text{ for } -1 \leq x \leq 1$$

Find the Fourier series of the function $f(x)$ (you may want to use integration by parts in the process).

(b) Let $g(x)$ be the periodic function of period $T = 2$ defined as

$$g(x) = \begin{cases} x, & 0 \leq x < 1, \\ 2 + (x-1)^2, & 1 \leq x < 2. \end{cases}$$

To what value does the Fourier series of $g(x)$ converge at the point $x = 1$? Justify.

(c) Write in your own words why are Fourier series useful.